Euclidean Geometry in Mathematical Olympiad

Problems and Solutions
Chapter 1: Angle Chasing

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1 Soal

1. Let $ABCDE$ be a convex pentagon such that $BCDE$ is a square with center O and $\angle A = 90°$. Prove that $\overline{AO}$ bisects $\angle BAE$.

2. Let $O = (0,0)$, $A = (0,a)$, and $B = (0,b)$, where $0 < a < b$ are reals. Let Γ be a circle with diameter $\overline{AB}$ and let P be any other point on Γ. Line PA meets the x-axis again at Q. Prove that $\angle BQP = \angle BOP$.

   Bay Area Mathematical Olympiad 1999/Problem 2

3. In cyclic quadrilateral $ABCD$, let I_1 and I_2 denote the incenters of $\triangle ABC$ and $\triangle DBC$, respectively. Prove that I_1I_2BC is cyclic.

4. Let ABC be a triangle. The incircle of $\triangle ABC$ is tangent to $\overline{AB}$ and $\overline{AC}$ at D and E respectively. Let O denote the circumcenter of $\triangle BCI$. Prove that $\angle ODB = \angle OEC$.

   China Girl Mathematical Olympiad 2012/Problem 5

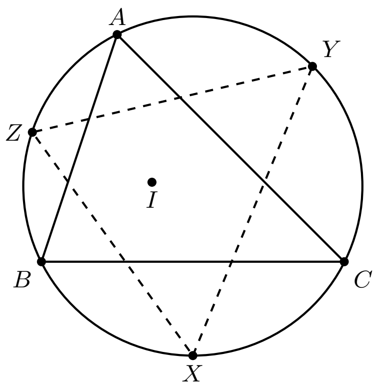
5. Let P be a point inside circle ω. Consider the set of chords of ω that contain P. Prove that their midpoints all lie on a circle.

   Canada 1991/Problem 3

6. Points E and F are on side $\overline{BC}$ of convex quadrilateral $ABCD$ (with E closer than F to B). It is known that $\angle BAE = \angle CDF$ and $\angle EAF = \angle FDE$. Prove that $\angle FAC = \angle EDB$.

   Russian Olympiad 1996

7. *Let ABC be an acute triangle incribed in circle Ω. Let X be the midpoint of the arc BC not containing A and define Y, Z similarly. Show that the orthocenter of XYZ is the incenter I of ABC.*

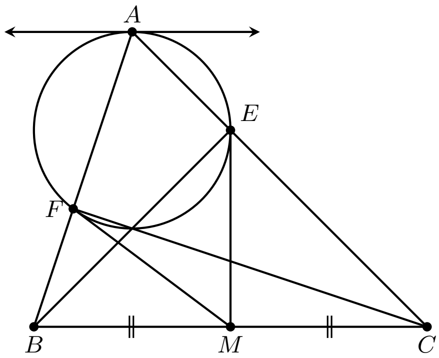


8. Points A, B, C, D, E lie on circle ω and point P lies outside the circle. The given points are such that
   (i) lines PB and PD are tangent to ω,
   (ii) P, A, C are collinear, and
   (iii) $\overline{DE} \parallel \overline{AC}$.

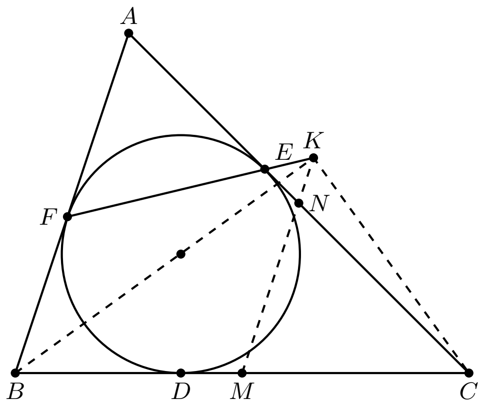
   Prove that $\overline{BE}$ bisects $\overline{AC}$.

   USA Junior Mathematical Olympiad/Problem 5

9. *Let ABC be an acute triangle. Let $\overline{BE}$ and $\overline{CF}$ be altitudes of $\triangle ABC$, and denote by M the midpoint of $\overline{BC}$. Prove that $\overline{ME}, \overline{MF}$, and the line through A parallel to $\overline{BC}$ are all tangents to (AEF).*



10. *The incircle of $\triangle ABC$ is tangent to $\overline{BC}, \overline{CA}, \overline{AB}$ at D, E, F, respectively. Let M and N be the midpoints of $\overline{BC}$ and $\overline{AC}$, respectively. Ray BI meets line EF at K. Show that $\overline{BK} \perp \overline{CK}$. Then shows K lies on line MN.*



11. The point O is situated inside the parallelogram $ABCD$ such that $\angle AOB + \angle COD = 180°$. Prove that $\angle OBC = \angle ODC$.

Canada 1997/Problem 4

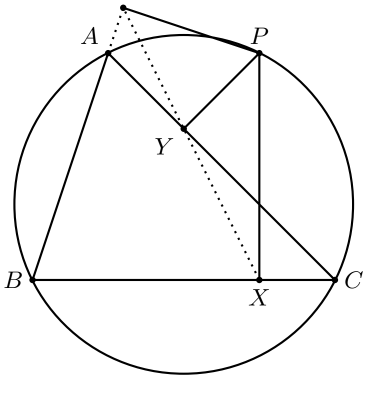
12. Let ABC be triangle with incenter I. A point P in the interior of the triangle satisfies

$$\angle PBA + \angle PCA = \angle PBC + \angle PCB.$$

Show that $AP \geq AI$ and that equality holds if and only if $P = I$.

International Mathematical Olympiad 2006/Problem 1

13. *Let ABC be a triangle and P be any point on (ABC). Let X, Y, Z be the feet of the perpendiculars from P onto lines BC, CA, and AB. Prove that X, Y, Z are collinear.*



14. Let $AXYZB$ be a convex pentagon inscribed in a semicircle of diameter AB. Denote by P, Q, R, S the feet of the perpendiculars from Y onto lines AX, BX, AZ, BZ, respectively. Prove that the acute angle formed by lines PQ and RS is half the size of $\angle XOZ$, where O is the midpoint of segment AB.

USA Mathematical Olympiad 2010/Problem 1

15. Let ABC be an acute triangle with orthocenter H, and let W be a point on the side $\overline{BC}$, between B and C. The points M and N are the feet of the altitudes drawn from B and C, respectively. ω_1 is the circumcircle of triangle BWN and X is a point such that $\overline{WX}$ is a diameter of ω_1. Similarly, ω_2 is the circumcircle of triangle CWM and Y is a point such that $\overline{WY}$ is a diameter of ω_2. Show that the points X, Y, and H are collinear.

International Mathematical Olympiad 2013/Problem 4

16. A circle has center on the side $\overline{AB}$ of the cyclic quadrilateral $ABCD$. The other three sides are tangent to the circle. Prove that $AD + BC = AB$.

International Mathematical Olympiad 1985/Problem 1

2 Soal dan Solusi

1. Let $ABCDE$ be a convex pentagon such that $BCDE$ is a square with center O and $\angle A = 90^\circ$. Prove that $\overline{AO}$ bisects $\angle BAE$.

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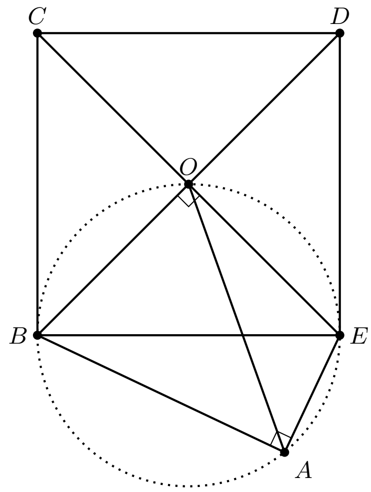

Karena $\angle BAE + \angle BOE = 180^\circ$, maka $ABOE$ siklis. Sehingga kita punya

$$\angle BAO = \angle BEO = 45^\circ \implies \angle BAO = 45^\circ.$$

Maka $\angle EAO = 45^\circ \implies \angle BAO = \angle EAO$. Jadi, terbukti bahwa $\overline{AO}$ membagi $\angle BAE$ dua sama besar.

2. Let $O = (0, 0), A = (0, a)$, and $B = (0, b)$, where $0 < a < b$ are reals. Let Γ be a circle with diameter $\overline{AB}$ and let P be any other point on Γ. Line PA meets the x-axis again at Q. Prove that $\angle BQP = \angle BOP$.

Bay Area Mathematical Olympiad 1999/Problem 2

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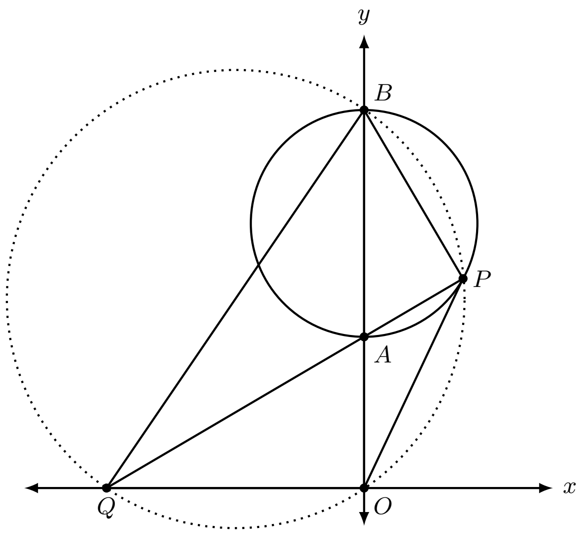

Karena Γ berdiameter $\overline{AB}$, maka $\angle BPA = 90°$. Di sisi lain, $\angle BOQ = 90°$. Karena $\angle BPQ = \angle BOQ$, maka $BPOQ$ siklis. Sehingga kita punya $\angle BQP = \angle BOP$.

3. In cyclic quadrilateral $ABCD$, let I_1 and I_2 denote the incenters of $\triangle ABC$ and $\triangle DBC$, respectively. Prove that $I_1 I_2 BC$ is cyclic.

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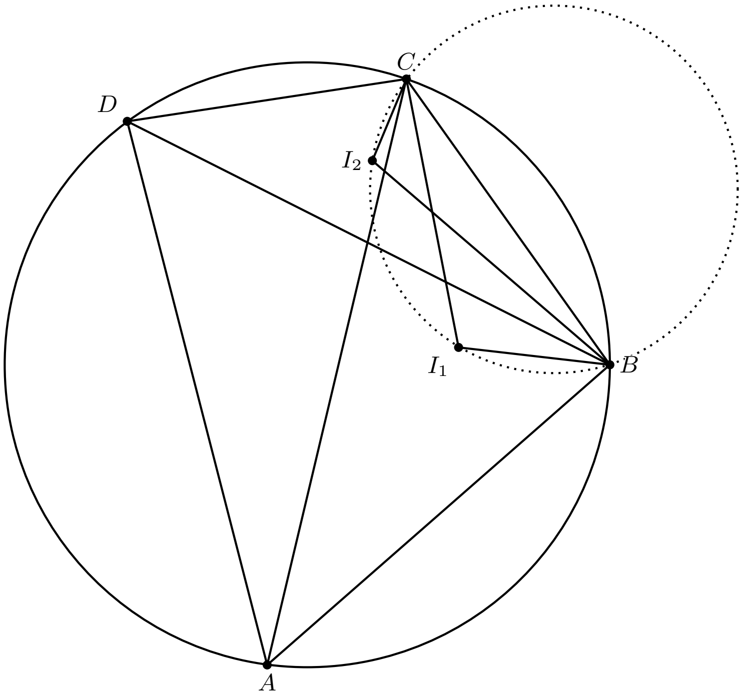

Perhatikan bahwa

$$\angle BI_1C = 90^\circ + \frac{\angle BAC}{2} = 90^\circ + \frac{\angle BDC}{2} = \angle BI_2C \implies \angle BI_1C = \angle BI_2C.$$

Akibatnya, $I_1 I_2 BC$ siklis.

4. Let ABC be a triangle. The incircle of $\triangle ABC$ is tangent to $\overline{AB}$ and $\overline{AC}$ at D and E respectively. Let O denote the circumcenter of $\triangle BCI$. Prove that $\angle ODB = \angle OEC$.

China Girl Mathematical Olympiad 2012/Problem 5

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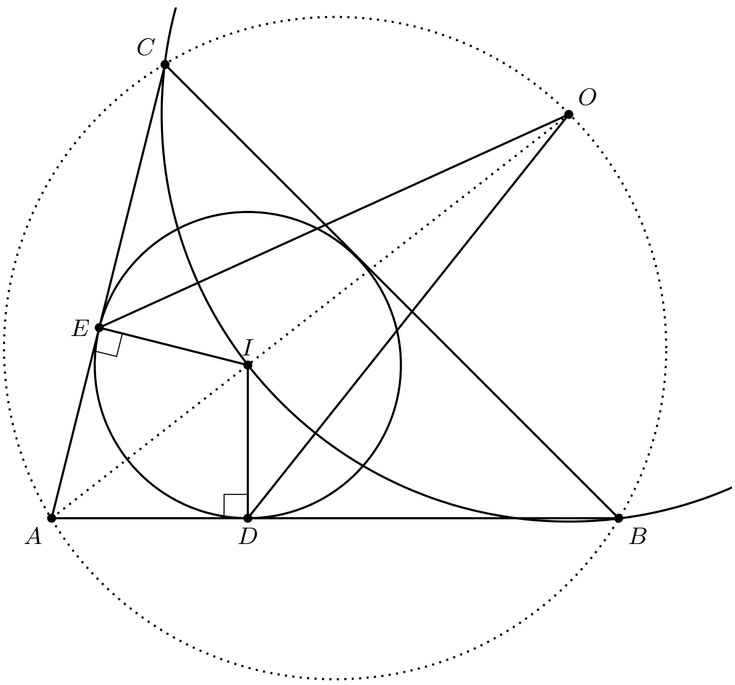
Misalkan I adalah titik pusat lingkaran dalam $\triangle ABC$. Dari **Incenter-Excenter Lemma**, maka O terletak di (ABC) dan A, I, O segaris.



Tinjau bahwa $\angle AEI = \angle ADI = 90^\circ$ dan panjang $IE = ID$, kita punya $\triangle EIA \cong \triangle DIA$. Maka panjang $AD = AE$ dan $\angle DAI = \angle EAI$. Karena $\angle EAO = \angle DAO$, panjang $AO = AO$, dan panjang $AD = AE$, maka $\triangle ADO \cong \triangle AEO$ (sisi-sudut-sisi). Maka $\angle ADO = \angle AEO \iff \angle ODB = \angle OEC$.

5. Let P be a point inside circle ω. Consider the set of chords of ω that contain P. Prove that their midpoints all lie on a circle.

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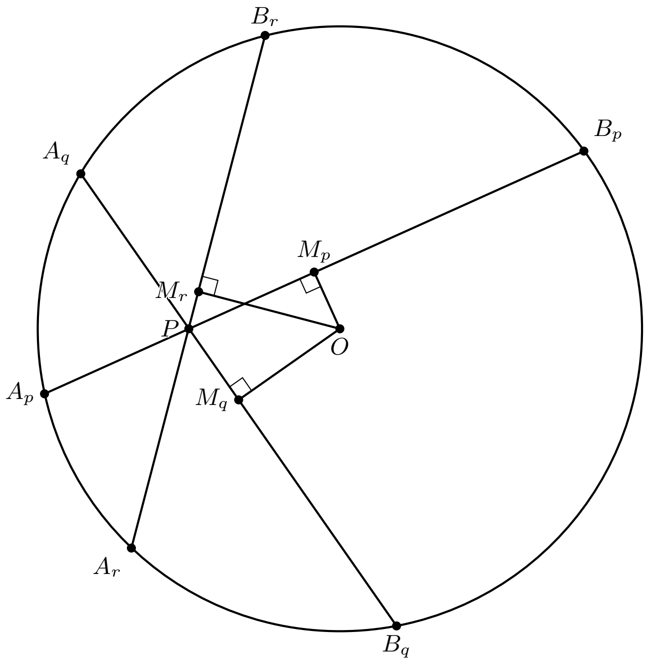
Misalkan $\overline{A_iB_i}$ menyatakan semua tali busur yang melalui titik P dan M_i sebagai titik tengahnya. Karena panjang $OA_P = OB_P$, $\angle OA_pM_p = \angle OB_pM_p$, dan panjang $A_pM_p = B_pM_p$, maka $\triangle OA_pM_p \cong OB_pM_p$ (sisi-sudut-sisi). Maka $\angle OM_pA_p = OM_pB_p = 90°$, analog untuk lainnya diperoleh $OM_i \perp A_iB_i$.



Karena $\measuredangle OM_pP = \measuredangle OM_rP$, akibatnya O, M_p, M_r, P siklis untuk setiap pasangan (p, r). Dengan kata lain, O, P, M_i akan terletak pada lingkaran yang sama untuk setiap i. Sehingga semua titik tengah dari tali busur $\overline{A_iB_i}$ akan terletak pada lingkaran yang sama.

6. Points E and F are on side $\overline{BC}$ of convex quadrilateral $ABCD$ (with E closer than F to B). It is known that $\angle BAE = \angle CDF$ and $\angle EAF = \angle FDE$. Prove that $\angle FAC = \angle EDB$.

Russian Olympiad 1996

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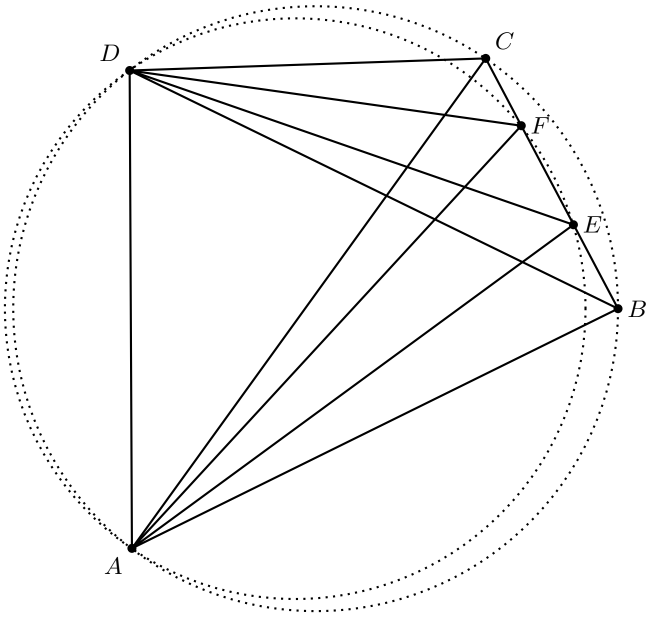


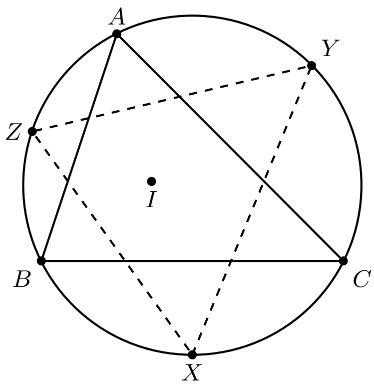
Karena $\angle EAF = \angle FDE$, akibatnya $AEFD$ siklis. Kita punya

$$\angle DAB = \angle DAE + \angle BAE = \angle DFC + \angle CDF = 180^\circ - \angle DCF.$$

Maka $\angle DAB + \angle DCF = 180^\circ$ yang berakibat $ABCD$ siklis. Maka

$$\begin{aligned} \angle BAC &= \angle BDC \\ \iff \angle BAE + \angle EAF + \angle FAC &= \angle CDF + \angle FDE + \angle EDB \\ \iff \angle FAC &= \angle EDB. \end{aligned}$$

7. *Let ABC be an acute triangle incribed in circle Ω. Let X be the midpoint of the arc BC not containing A and define Y, Z similarly. Show that the orthocenter of XYZ is the incenter I of ABC.*



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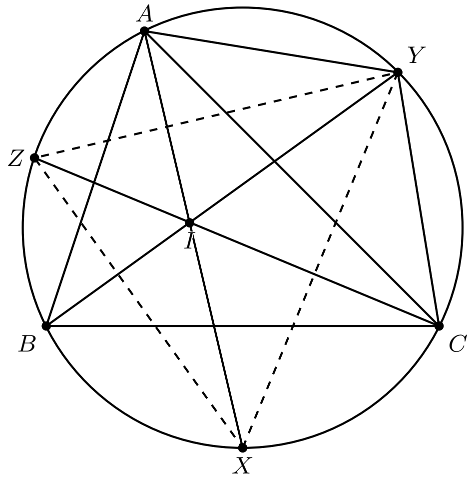
Karena panjang $AY = YC$, maka $\angle YAC = \angle YCA$. Sehingga $\angle YBA = \angle YCA = \angle YAC = \angle YBC \implies \angle YBA = \angle YBC$. Maka B, I, Y segaris. Secara analog, A, I, X segaris dan C, I, Z juga segaris. Akan kita buktikan bahwa $\angle BYX + \angle ZXY = 90^\circ$.



Misalkan $\angle A = 2\alpha, \angle B = 2\beta$, dan $\angle C = 2\gamma$. Maka $\angle A + \angle B + \angle C = 180^\circ \iff \alpha + \beta + \gamma = 90^\circ$. Karena $BAYX$ siklis, maka $\angle BYX = \angle BAX = \alpha$. Secara analog, kita peroleh $\angle ZXA = \angle ZCA = \gamma$ dan $\angle AXY = \angle ABY = \beta$. Maka $\angle BYX + \angle ZXY = \alpha + \beta + \gamma = 90^\circ$. Sehingga $BY \perp XZ$. Secara analog, kita peroleh $AX \perp YZ$ dan $CZ \perp XY$. Maka I merupakan titik tinggi dari $\triangle XYZ$.

8. Points A, B, C, D, E lie on circle ω and point P lies outside the circle. The given points are such that

   (i) lines PB and PD are tangent to ω,

   (ii) P, A, C are collinear, and

   (iii) $\overline{DE} \parallel \overline{AC}$.

   Prove that $\overline{BE}$ bisects $\overline{AC}$.

USA Junior Mathematical Olympiad/Problem 5

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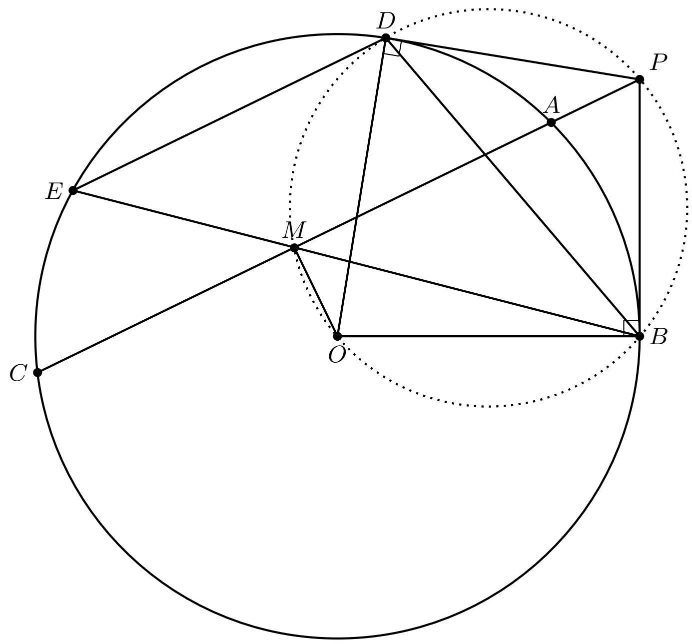
W.L.O.G. $PA < PC$. Misalkan O merupakan titik pusat ω dan $M = BE \cap AC$.

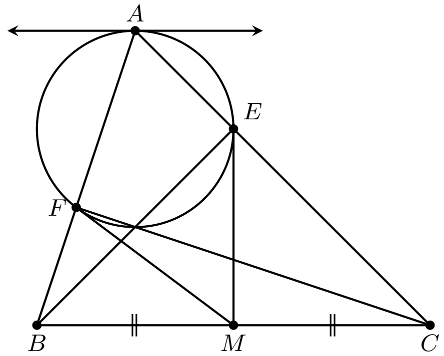


Karena panjang $OB = OD$, maka $\angle OBD = \angle ODB$. Karena $AC \parallel DE$, maka

$$\angle BMA = \angle BED = \frac{\angle BOD}{2} = \frac{180^\circ - 2\angle ODB}{2} = 90^\circ - \angle ODB = \angle PDB.$$

Karena $\angle PDB = \angle BMA = \angle BMP \implies \angle PDB = \angle BMP$, akibatnya $BMDP$ siklis. Di sisi lain, $\angle OBP + \angle ODP = 180^\circ$, maka $OBPD$ siklis. Jadi, $BOMDP$ siklis. Akibatnya, $\angle OMP = \angle ODP = 90^\circ \implies OM \perp AC$. Maka M merupakan titik tengah dari $\overline{AC}$.

9. *Let ABC be an acute triangle. Let $\overline{BE}$ and $\overline{CF}$ be altitudes of $\triangle ABC$, and denote by M the midpoint of $\overline{BC}$. Prove that $\overline{ME}, \overline{MF}$, and the line through A parallel to $\overline{BC}$ are all tangents to (AEF).*



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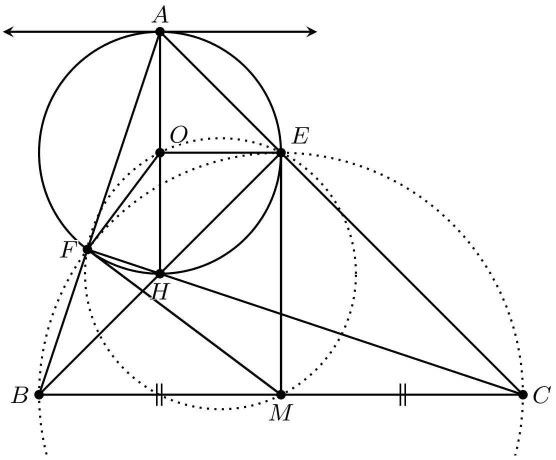
Misalkan O dan H berturut-turut merupakan titik tinggi $\triangle ABC$ dan pusat (AEF). Karena $\angle AEH + \angle AFH = 180^\circ$, maka $AEHF$ siklis dan $\overline{AH}$ sebagai diameter (AEF).

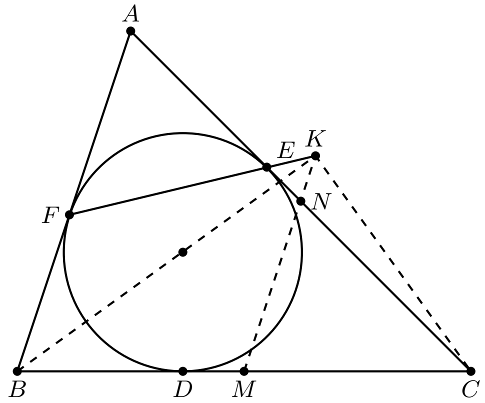


Perhatikan bahwa $\angle BFC = \angle BEC = 90^\circ$, maka $BFEC$ siklis dan $\overline{BC}$ sebagai diameter $(BFEC)$. Maka M merupakan titik pusat $(BFEC)$. Kita punya

$$\angle FME = 2\angle FCE = 2\angle FCA = 180^\circ - 2\angle FAC = 180^\circ - 2\angle FAE = 180^\circ - \angle FOE.$$

Maka $\angle FME + \angle FOE = 180^\circ$ sehingga $FMEO$ siklis. Karena panjang $\overline{MF} = \overline{ME}$ dan $\overline{OF} = \overline{OE}$, maka $FMEO$ merupakan layang-layang. Sehingga $\angle MFO = \angle MEO = 90^\circ$ yang berarti MF dan ME merupakan garis singgung (AEF). Selain itu, karena $AH \perp BC$, maka garis yang sejajar dengan BC akan tegak lurus dengan $\overline{AH}$. Artinya, garis tersebut merupakan garis singgung (AEF).

10. *The incircle of $\triangle ABC$ is tangent to $\overline{BC}, \overline{CA}, \overline{AB}$ at D, E, F, respectively. Let M and N be the midpoints of $\overline{BC}$ and $\overline{AC}$, respectively. Ray BI meets line EF at K. Show that $\overline{BK} \perp \overline{CK}$. Then shows K lies on line MN.*



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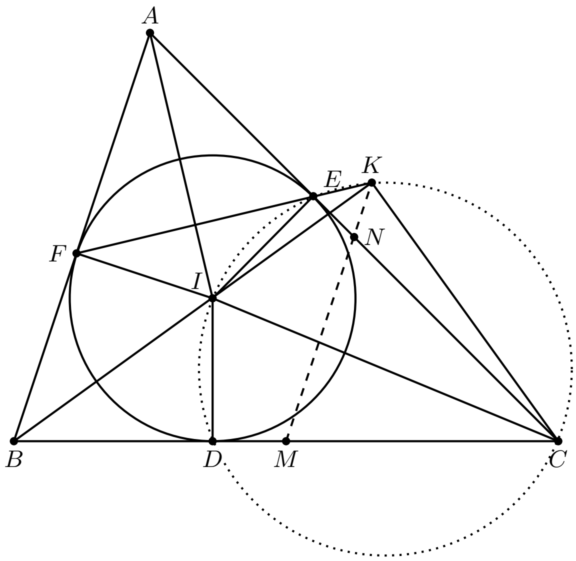

Misalkan I titik bagi $\triangle ABC$. Perhatikan bahwa panajng $IE = IF, IA = IA$, dan $\angle IAF = \angle IAE$, maka $\triangle IFA \cong \triangle IEA$. Sehingga panjang $AF = AE$. Maka $\angle AFE = \angle AEF = 90^\circ - \frac{\angle FAE}{2} = 90^\circ - \frac{\angle A}{2}$. Maka $\angle BFK = 90^\circ + \frac{\angle A}{2}$. Dari $\triangle BFK$, kita punya

$$\angle IKE = \angle BKF = 180^\circ - \angle BFK - \angle FBK = 90^\circ - \frac{\angle A + \angle B}{2} = \frac{\angle C}{2} = \angle ICE.$$

Karena $\angle IKE = \angle ICE$, maka $ICKE$ siklis. Sehingga kita peroleh $\angle BKC = \angle IKC = \angle IEC = 90^\circ$. Jadi, $BK \perp KC$. Dari **Midpoint Theorem**, $MN \parallel AB$. Untuk membuktikan K, M, N segaris, hal ini ekuivalen dengan membuktikan $KM \parallel AB \iff \angle KMC = \angle ABC$. Karena $\angle BKC = 90^\circ$, maka $\overline{BC}$ merupakan diameter (BKC) yang berarti M pusat (BKC). Maka

$$\angle KMC = 2\angle KBC = 2 \cdot \frac{\angle B}{2} = \angle B = \angle ABC \implies \angle KMC = \angle ABC$$

seperti yang ingin dibuktikan.

11. The point O is situated inside the parallelogram $ABCD$ such that $\angle AOB + \angle COD = 180^\circ$. Prove that $\angle OBC = \angle ODC$.

Canada 1997/Problem 4

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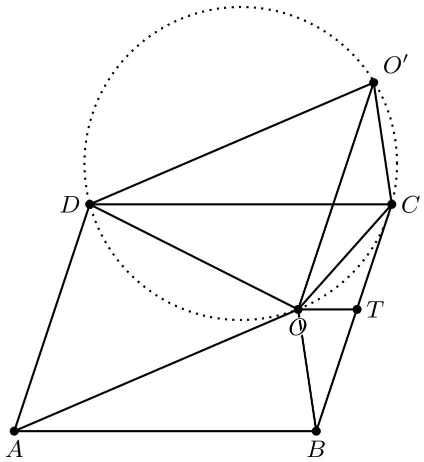
Misalkan O' terletak di luar $ABCD$ sehingga $\triangle AOB \cong \triangle DO'C$.



Maka $\angle DO'C + \angle DOC = \angle AOB + \angle DOC = 180^\circ$. Akibatnya, $DOCO'$ siklis. Misalkan garis yang melalui O dan sejajar AB memotong BC di T. Misalkan juga $\angle OAB = \angle O'DC = a, \angle OBA = \angle O'CD = b$, dan $\angle OCD = c$. Kita punya $\angle DOO' = \angle DCO' = b$ dan $\angle COO' = \angle CDO' = a$. Karena $OT \parallel DC$ dan $OT \parallel AB$, maka $\angle TOC = \angle DCO = c$ dan $\angle TOB = \angle ABO = b$. Kita punya $\angle BOC = \angle O'CO = b + c$, panjang $OB = O'C$, dan panjang $OC = OC$, maka $\triangle OBC \cong \triangle CO'O$ (sisi-sudut-sisi). Maka $\angle OBC = \angle CO'O = \angle CDO \implies \angle OBC = \angle CDO$.

> **Remark.** Pembuktian $\angle BOC = \angle O'CO$ dapat diperoleh dengan membuktikan $BO \parallel CO'$. Tinjau
>
> $$\angle OBC + \angle O'CB = \angle ABC - \angle ABO + \angle O'CD + \angle DCB = \angle ABC + \angle DCB = 180^\circ.$$
>
> Akibatnya, $BO \parallel CO'$.

12. Let ABC be triangle with incenter I. A point P in the interior of the triangle satisfies

$$\angle PBA + \angle PCA = \angle PBC + \angle PCB.$$

Show that $AP \geq AI$ and that equality holds if and only if $P = I$.

International Mathematical Olympiad 2006/Problem 1

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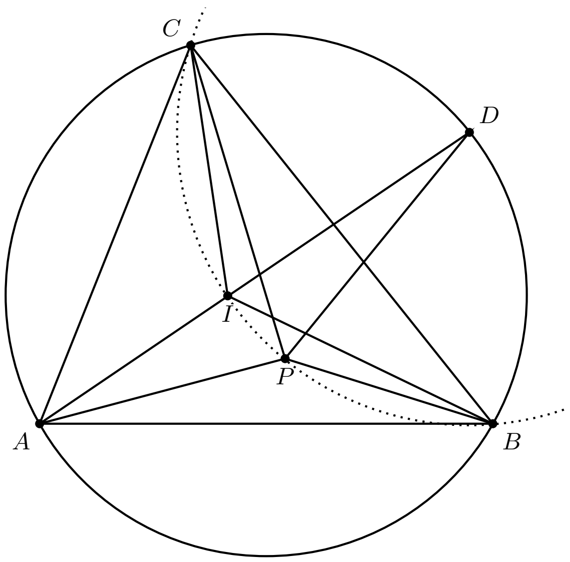
Misalkan $AI \cap (BIC) = D$. Dari **Incenter-Excenter Lemma**, maka D pusat (BIC).



Misalkan $\angle PBA + \angle PCA = \angle PBC + \angle PCB = x$. Maka $2x = \angle ABC + \angle BCA$ dan diperoleh

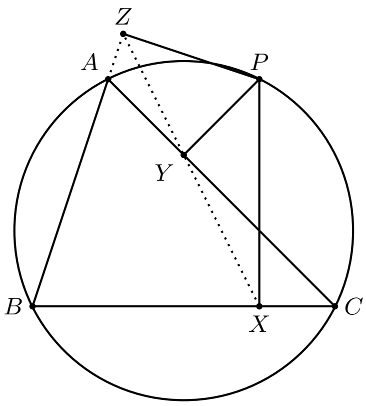
$$\angle BIC = 90^\circ + \frac{\angle A}{2} = 90^\circ + \frac{180^\circ - 2x}{2} = 180^\circ - x.$$

Sedangkan, kita punya juga $\angle BPC = 180^\circ - \angle PBC - \angle PCB = 180^\circ - x \implies \angle BPC = \angle BIC$. Akibatnya, B, P, I, C siklis. Maka panjang $DP = DI$. Dari ketaksamaan segitiga dari $\triangle APD$,

$$AD \geq AP + PD \iff AI + ID \geq AP + PD \iff AI \geq AP$$

di mana kesamaan terjadi jika dan hanya jika $P = I$.

13. *Let ABC be a triangle and P be any point on (ABC). Let X, Y, Z be the feet of the perpendiculars from P onto lines BC, CA, and AB. Prove that X, Y, Z are collinear.*



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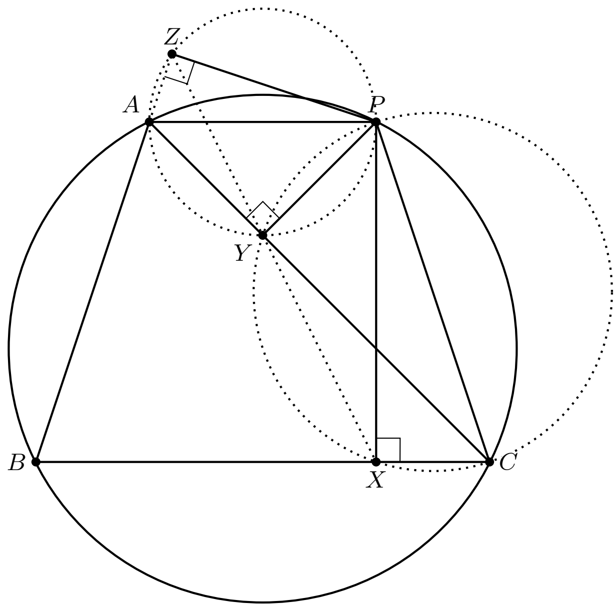

Karena $\measuredangle PYA = \measuredangle PZA$ dan $\measuredangle PYC = \measuredangle PXC$, maka $PZAY$ dan $PYXC$ masing-masing siklis. Kita punya

$$\measuredangle PYZ = \measuredangle PAZ = \measuredangle PCB = \measuredangle PCX = \measuredangle PYX \implies \measuredangle PYZ = \measuredangle PYX.$$

Maka X, Y, Z kolinear.

14. Let $AXYZB$ be a convex pentagon inscribed in a semicircle of diameter AB. Denote by P, Q, R, S the feet of the perpendiculars from Y onto lines AX, BX, AZ, BZ, respectively. Prove that the acute angle formed by lines PQ and RS is half the size of $\angle XOZ$, where O is the midpoint of segment AB.

USA Mathematical Olympiad 2010/Problem 1

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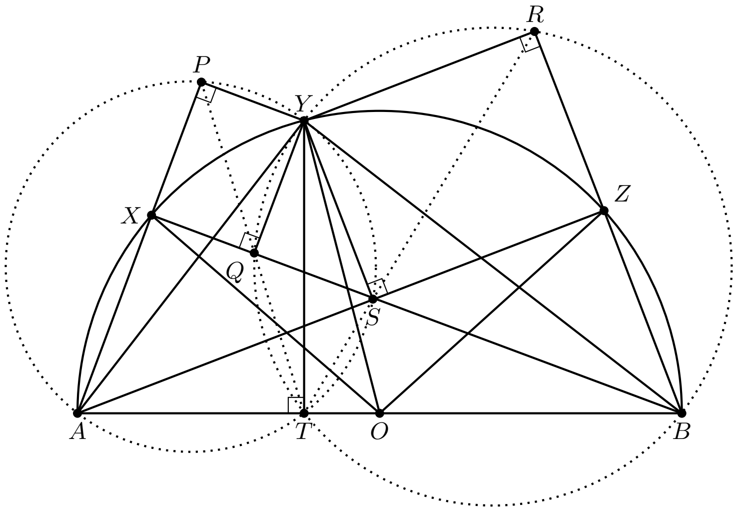
Misalkan titik T pada $\overline{AB}$ sehingga $AB \perp YT$. Jelas O pusat setengah lingkaran berdiameter AB.



Dari **Simson Line Lemma**, maka P, Q, T segaris dan R, S, T segaris. Tinjau bahwa $\measuredangle APY = \measuredangle ATY$, maka $ATYP$ siklis. Secara analog, $BTYR$ siklis. Kita punya

$$\begin{aligned}\angle PTR &= \angle PTY + \angle RTY \\ &= \angle PAY + \angle RBY \\ &= \angle XAY + \angle ZBY \\ &= \frac{\angle XOY}{2} + \frac{\angle ZOY}{2} \\ &= \frac{\angle XOZ}{2}\end{aligned}$$

seperti yang ingin dibuktikan.

15. Let ABC be an acute triangle with orthocenter H, and let W be a point on the side $\overline{BC}$, between B and C. The points M and N are the feet of the altitudes drawn from B and C, respectively. ω_1 is the circumcircle of triangle BWN and X is a point such that $\overline{WX}$ is a diameter of ω_1. Similarly, ω_2 is the circumcircle of triangle CWM and Y is a point such that $\overline{WY}$ is a diameter of ω_2. Show that the points X, Y, and H are collinear.

International Mathematical Olympiad 2013/Problem 4

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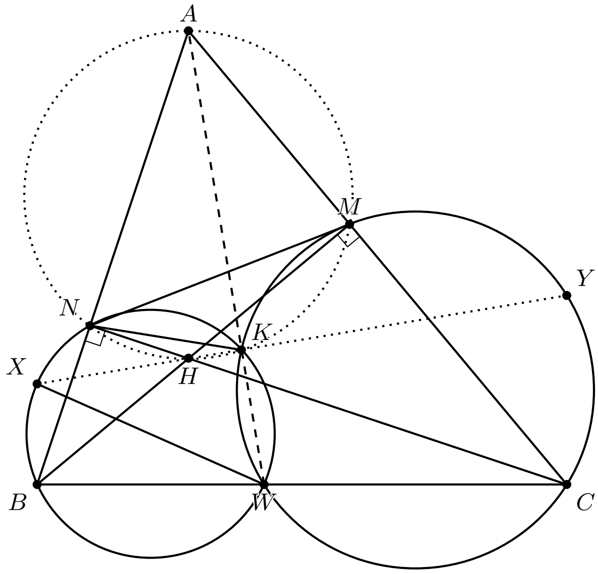
Misalkan $\omega_1 \cap \omega_2 \in \{K, W\}$. Karena WX diameter ω_1, maka $\angle WKX = 90^\circ$. Dengan cara sama, diperoleh $\angle WKY = 90^\circ$ dan diperoleh $\angle WKX + \angle WKY = 180^\circ$. Maka X, K, Y segaris.



Dari **Miquel Point Theorem**, kita peroleh bahwa $ANKM$ siklis. Karena $\angle BNC = \angle BMC$, maka $BNMC$ siklis. Kita punya

$$\measuredangle NBW = \measuredangle NBC = \measuredangle NMA = \measuredangle NKA \implies \measuredangle NBW = \measuredangle NKA.$$

Maka W, K, A segaris. Karena $\angle ANH + \angle AMH = 180^\circ$, maka $ANHM$ siklis. Jadi, kita simpulkan A, N, H, K, M siklis. Maka $\measuredangle AKH = \measuredangle AMH \implies \angle AKH = 90^\circ$. Karena W, K, A segaris, maka $\angle WKH = 90^\circ \implies \angle WKH = \angle WKX$ yang berarti K, H, X segaris. Karena X, K, Y juga segaris, kita simpulkan bahwa X, Y, H segaris.

> **Remark.** Pembuktian A, K, W segaris dapat menggunakan Radical Axis Theorem. Tinjau bahwa $BNMC$ siklis karena $\angle BMC = \angle BNC$. Dapat ditinjau bahwa AB merupakan radical axis ω_1 dan $(BNMC)$, AC merupakan radical axis ω_2 dan $(BNMC)$, dan KW merupakan radical axis ω_1 dan ω_2. Maka AB, AC, KW akan berpotongan di satu titik, yaitu titik A. Soal ini menjadi soal favorit saya di bab ini.

16. A circle has center on the side $\overline{AB}$ of the cyclic quadrilateral $ABCD$. The other three sides are tangent to the circle. Prove that $AD + BC = AB$.

International Mathematical Olympiad 1985/Problem 1

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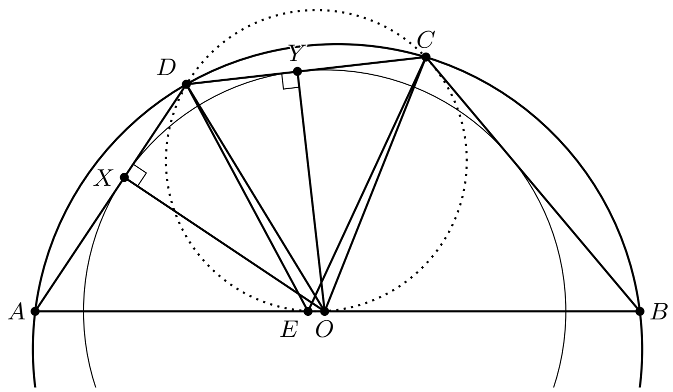
Misalkan titik E pada $\overline{AB}$ sehingga panjang $AD = AE$ serta O adalah pusat lingkaran yang menyinggung ketiga sisi dari segiempat $ABCD$. Sehingga sekarang ekuivalen dengan membuktikan panjang $BE = BC$ atau ekuivalen pula dengan membuktikan $\angle BCE = \angle BEC$. Misalkan lingkaran O menyinggung $\overline{AD}$ dan $\overline{DC}$ berturut-turut di titik X dan Y.



Karena panjang $OX = OY$ dan $\angle OXD = \angle OYD$, maka $\triangle OXD \cong \triangle OYD$. Sehingga $\angle ODX = \angle ODY$ yang berarti OD garis bagi $\angle ADC$. Secara analog, OC garis bagi $\angle BCD$. Karena panjang $AD = AE$, maka $\angle ADE = \angle AED$. Karena $ABCD$ siklis, maka

$$2\angle DCO = \angle DCB = 180^\circ - \angle DAB = 180^\circ - \angle DAE = \angle AED + \angle EDA = 2\angle AED$$

dan diperoleh $\angle DCO = \angle AED$, maka E, O, C, D siklis. Kita punya

$$\angle CEB = \angle CEO = \angle CDO = \frac{\angle CDA}{2} = \frac{180^\circ - \angle CBA}{2} = \frac{180^\circ - \measuredangle CBE}{2} = \frac{\angle CEB + \angle BCE}{2}$$

yang menyimpulkan $\angle CEB = \angle BCE$ seperti yang ingin dibuktikan.